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A Method of Measuring Inequality Within a Selection Process

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Abstract

To explain the inequalities in access to a discrete good *G* across two populations, or across time in a single national context, it is necessary to distinguish, for each population or period of time, the effect of the diffusion of *G* from that of unequal outcomes of underlying micro-social processes. The inequality of outcomes of these micro-social processes is termed inequality within the selection process. We present an innovative index of measurement that captures variations in this aspect of inequality of opportunity and is insensitive to margins. We applied this index to the analysis of inequality of educational opportunity by exploring the effects of the British 1944 Education Act, of which various accounts have been offered. The relationships between the measure of inequality within a selection process presented and classical measures of inequality of opportunity are analyzed, as well as the benefits of using this index with regard to the insight it provides for interpreting data.

Keywords

inequality of opportunity, selection process, inequality measures, margin insensitivity, 1944 Education Act, odds ratio

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Introduction

The methodological problem at issue here arises in any field where comparing access to a discrete good, *G*, requires explanations of the social processes generating observed inequality, such as labor discrimination, schooling inequalities, social mobility, urban segregation, and so on.

When we account for situations of unequal access to a specific good *G* across two populations, or across time in a single national context, we need to differentiate the effect of social processes from the effect of margins—the distribution of social subgroups in the population and the overall rate of access to the good *G*—on the observed inequalities. In this respect, much has been written on the importance of margin insensitivity of indices in comparative studies, but there has been a tendency to confuse a measurement of inequality insensitive to marginal distributions with inequality of social processes irrespective of the margins' role. A margin-insensitive index is independent of the value of the margins regarding the specific aspect of inequality it captures, while assessing inequality of social processes irrespective of the role of the margins relates to an aspect of inequality of opportunity that none of the conventional indices account for. To address this issue, we rely on the index of inequality developed by Bulle (2009), which permits a comparison of the results of the selection process for access to a discrete good *G*—that is, the outcomes of the micro-social processes that underlie access to *G*, irrespective of the opening up of access to *G*.

To support our argument, we use an example: an analysis of the effect of the British 1944 Education Act on inequality of access to a grammar school education. We compare the assessments provided by some conventional measures of inequality and illustrate the limitations of their accounts of the observed trends. We then outline the case for an alternative measurement method and present the measure developed by Bulle (2009). Next, we analyze the relationships between this measure and various indices of inequality of opportunity. Returning to the 1944 example, we show how using this new measurement method improves the analysis and provide an interpretation of the new results.

Measuring Inequality of Educational Opportunity: The Example of the British 1944 Education Act

The 1944 Act

From 1902, and for nearly half a century, pupil selection for secondary education in British schools was based on a competitive examination taken at the age of 11, with an important place given to intelligence tests. Pupils who passed the examination obtained a free place in a grammar school; pupils who failed the examination were only able to follow an equivalent course of studies in a private school.[1] The 1944 Education Act extended free education to all state secondary schools, raised the school-leaving age from 14 to 15 (effective from 1947), and introduced the tripartite system dividing the secondary system into three types of schools, namely, grammar schools, secondary modern schools, and secondary technical schools. The kind of secondary school education students received depended mainly on the results of the "11-plus" examination—which was similar to the old Special Place examination to which it replaced. Even if the 1944 Act is "widely regarded as a landmark of social change" (Blackburn and Marsh 1991:508) making free secondary education available to all, and also offering a greater number of free places in selective schools, the meritocratic selection process varied only slightly from the system it superseded.[2]

The effect of these changes on the inequality of access to selective schools in England has been studied by a number of sociologists using various data sets, measures, and models. The pursuit of improvements in measurement tools has been unending, with the objective of understanding accurately the role in educational inequality of the main factors investigated here, that is, selection processes and the availability of places in selective schools.

Analyses of the Effects of the 1944 Education Act

In an article that is a standard reference on the issue, Little and Westergaard (1964) considered the evolution of inequality of access to lengthy secondary education (i.e., in grammar and independent schools) for English children in the first half of the twentieth century. Their data on boys' and girls' secondary schooling were derived from the following two sources: Floud's (1954) analysis of a survey conducted in July 1949—a random sample of 10,000 English men and women aged 18 years and over (born before the Second World War)—and the Crowther Report's (1959) sample of national service recruits (cf. Table 1).[3] Little and Westergaard offered a mixed account, pointing out that different results were obtained depending on whether the analysis was based on the chances of gaining access to a selective school—which were multiplied by 10 for children in the lowest social group but by 1.7 for those of the upper group—or on the risk of not gaining access to them—which reduced by nearly half for children from homes in the upper social group but by barely a tenth for children from the lowest social group. Everything depends, the authors concluded, on "the relative weight one attaches to

Table 1. Proportions in Different Classes Obtaining Education of a Grammar School Type[a] by Birth Cohort.

% Boys and Girls	Born Pre-1910	Born 1910–1919	Born 1920–1929	Born Late 1930s
Prof./ Managerial	37	47	52	62
Other nonmanual and skilled manual	7	13	16	20
Semi- and unskilled	1	4	7	10

Source: Little and Westergaard (1964: table 4, 309).
[a]Grammar and independent schools.

the proportion achieving, as compared with the proportion who fails to achieve selective secondary schooling." But, they explained, the effect of the 1944 Act on these inequalities was still not clear; the reduction of social inequalities that followed the Act was a continuation of a long-term, gradual trend.

Comparing data on the evolution of inequality of opportunity in different countries with the data for Great Britain published by Little and Westergaard—because such data provided a long-term picture of the evolutions at issue, Boudon (1974:143-58) made the assumption that a similar structural change occurred in the expanding educational systems of industrialized societies. This structural change in the inequality of opportunity underpins the general explanatory model of expansion of educational systems that Boudon developed.[4] Also referring to the data published by Little and Westergaard (1964:309), Combessie (1984) concluded that there was an irreducible diversity of accounts of inequality of opportunity, given the various foci of the measures used.

Blackburn and Marsh (1991) reopened the case, this time on the basis of data published by Halsey, Heath, and Ridge (1980), derived from the Oxford Mobility Study, a representative sample of English and Welsh men interviewed in 1972 (cf. Table 2) divided into four 10-year birth cohorts, two entering secondary education before the 1944 Act and two afterward. The original analysis was restricted to men between the ages of 20 and 59 who received their secondary education in England and Wales. Blackburn and Marsh noted that no clear conclusions about the effect of the 1944 Act itself were drawn by Halsey et al. and that, according to Heath's words in subsequent works, the study showed that nothing had changed over the years. Blackburn and Marsh reconsidered the same data and compared accounts based on various measures of inequality, particularly the segregation index

Table 2. Attendance of Boys at Selective Schools[a] by Class and Birth Cohort.

% (Eff. Class)	Born 1913–1922	Born 1923–1932	Born 1933–1942	Born 1943–1952
Class 1	65 (172)	78 (204)	78 (238)	65 (430)
Class 2	37 (595)	44 (568)	42 (576)	38 (744)
Class 3	20 (1,084)	26 (1,102)	28 (1,050)	21 (1,153)
Total	29 (1,851)	37 (1,874)	39 (1,864)	35 (2,327)

Source: Blackburn and Marsh (1991: table 1, 511).
[a]Including state sector technical and grammar schools and all private schools.

they developed, which is insensitive to marginal distributions: the marginal matching coefficient (MM).

It should be noted that Blackburn and Marsh departed from Halsey et al. in their approach to class categorization. In the original analysis, respondents were assigned to one of the three classes on the basis of paternal occupation when the children were aged 14 years: a "service" class of professionals and managers at the top, a "working" class of manual workers at the bottom, and an "intermediate" class in between. Blackburn and Marsh recoded paternal occupations using an index of social advantage—based on social networks analysis—and then divided the occupational scale into class sizes within each 10-year cohort to produce the class margins reported by Halsey et al.

As shown in Table 2, the first three birth cohorts saw their access to selective forms of secondary education increase; but for the last, access declined as population growth outstripped the rise in places. Blackburn and Marsh noted that different classical measures of association between educational and social stratification give different accounts of the evolution of educational opportunity (see Table 3) for the top and bottom classes (given in Table 2). Looking at the ratio of proportions *RP*, educational opportunity diminished in the first three periods and rose in the last; according to the phi coefficient, Φ, opportunity rose throughout the period, whereas the odds ratio, Ω, and the difference of proportions, *d*, indicated that they rose initially, before falling after the Act.

If one wants to know what intrinsic changes in inequality of opportunity may have occurred, the problem is, as Blackburn and Marsh noted, how to disentangle such changes from the effects of variation in margins. Class size and school place margins constrain the range of *RP, d*,[5] and Φ.[6] Because there were more selective places than children in class 1 (as defined in Table 2), children from other classes had to be selected. Besides, the *RP* measure is

Table 3. Different Summaries of the Relationships between Class and Selection, for Classes 1 and 3, in the Four Cohorts.

Cohort	Ratio of proportions RP	Odd ratios (cross product) Ω	Difference of proportions *d*	Phi Φ
1913–1922	3.27	7.50	0.452	0.354
1923–1932	3.01	10.13	0.521	0.399
1933–1942	2.82	9.33	0.504	0.405
1943–1952	3.09	6.99	0.440	0.416

Source: Blackburn and Marsh (1991: table 3, 517).

incomplete because it only takes into account the selected and not the nonselected children. Using the cross product or odds ratio (Ω)[7] avoids these limitations: It takes both selected and nonselected children into account and is insensitive to margins variation.

As we will see in the next section, margin insensitivity notwithstanding, Ω cannot capture the changes in inequality of opportunity we are looking for to support explanation of the evolutions under consideration—that is, changes in inequality regarding the effect of the micro-social processes generating access to the good *G* at stake—for instance, a grammar school education—irrespective of the availability of places in grammar schools.

Measuring Inequality Within a Selection Process

The Issue of Margin Insensitivity

The degree of inequality of access to a discrete good *G* measured by an index that is insensitive to marginal distributions should not be limited either by the proportion of individuals in the population gaining access to *G* or by the relative size of the various social groups. Consequently, these proportions, or the relationships between these proportions, should not constrain the index values, such that in each context defined by the contingency table's margins, the same degree of inequality may be observed. These conditions that exclude any structural artifact from the comparison of the degree of inequality in different social contexts are, of course, extremely restrictive.

For instance, we saw previously that chances of gaining access to a selective school were multiplied by 10 for children in the lowest social group and by 1.7 for those of the upper social group. But, as Boudon (1974:143) notes, the rate of access of children from the upper social group could not be multiplied by a coefficient higher than $\frac{100\%}{37\%} = 2.7$, whereas the rate of access of children from the lowest social group could be multiplied by $\frac{100\%}{1\%} = 100$. These limitations imposed by the margins make the meaning of the evolution of inequality of opportunity problematic here and can only be overcome by the use of a margin-insensitive index.

Margin insensitivity has raised three main issues in the sociological literature, namely, its importance for comparative analysis (for elements of this debate, see Hellevik 2000; Marshall and Swift 1999, 2000; Ringen 2000), the specificity of the odds ratio—today widely used as measure of association, particularly in log-linear modeling (see, for instance, Breen and Jonsson 2005; Hout and DiPrete 2006; Lucas 2010; Shavit, Arum, and Gamoran 2007; Shavit and Blossfeld 1993) and the link between odds ratios and what we refer as the selection process.

First, the importance of margin insensitivity for comparative analysis depends on which aspect of inequality is being analyzed. At issue is whether the value of the margins may restrict the range of the inequality under consideration. If this is the case, margins are an integral part of the comparison—for example, in concentration measured by the Gini coefficient, where widespread diffusion of *G* counteracts seizure by an advantaged population. However, indices that show, for example, that inequality tends to decrease as the good *G* (here, the level of education) spreads among the population, can be pertinent when *G* is an ordinary consumption good, but are less so when the meaning of *G*—or else the value of *G*—has an important relative component.[8] This is why the concept of inequality of opportunity is more generally used within the sphere of education and tends in particular to be applied to inequality of access to the highest levels.

Margin insensitivity is a necessary property of measures of inequality when seeking to make comparisons that are not restricted in an artifactual way by the structures that represent the margins. The problem has been solved for measurement of association (inequality of opportunity)—the solution relies on the odds ratio—and for measurement of segregation—the solution relies on the MM coefficient.

The second problem addressed in the sociological literature concerns the specificity of the odds ratios with regard to margin insensitivity. In the classical meaning, insensitivity to marginal distributions requires that an index does not change if any row or column of the contingency table is multiplied throughout by a constant. According to this meaning, a margin-insensitive index must be based on ratios, so that changes in margins do not affect the meaning of the magnitude of the index in terms of relative inequality—that is, the index value remains stable if proportionalities are respected. This specific condition, which guarantees that the broad condition stated previously will also be fulfilled, applies to odds ratios and measures based on them: Thanks to the way odds ratios are calculated, it is possible to preserve proportionalities so that the same degree of inequality may be observed, whatever the margins. But the reverse is not true, that is, insensitivity to marginal variations does not imply this property (Blackburn, Siltanen and Jarman, 1995:325; Hellevik 2000, 2007). Nevertheless, the specific form of margin insensitivity represented by odds ratios has usually been considered as the general one, and the fact that odds ratios are insensitive to the contingency table's margins only with respect to their own object—for instance, intrinsic association between school attainment and socioeconomic stratification—has been obscured.

The third and related issue concerns the relationship between the odds ratio, the so-called allocation mechanism, which designs the social mechanisms leading to access to a good *G*, and the selection process results that identify, as defined, the outcomes of the micro-social processes that underlie access to *G*, irrespective of the opening up of access to *G*. As stated in the Introduction, a problem arises when an explanation of observed inequalities is sought. Explanatory accounts of observed inequalities represent a complementary approach to inequality measurement and refer to the formulation of assumptions concerning the generative mechanisms that underlie the observed inequalities.

The generative mechanisms invoked in sociological explanations describe causal relationships situated at a lower level (individual, here) of analysis in order to explain observed phenomena at a higher level (e.g., groups); the starting point for understanding these social mechanisms is data on the main entities (groups), from which assumptions are necessarily developed (Stinchcombe 1991). Regarding inequality of access to a good *G*, the formulation of explanatory assumptions—that is, causal relationships—necessitates separating the effect of the micro-social processes generating unequal access to a good *G* from the opening up of access to *G*.

A margin-insensitive index does not *ipso facto* capture the effects of the selection process as defined. The oft-cited assertion that "statistical models that measure the association between school continuation and social background, net of the marginal distribution of schooling, are sensitive to changes in the principles by which schooling is allocated and not to changes in the dispersion of the schooling distribution" (Mare 1981:83) shows the confusion existing about this issue (see Logan 1996). Odds ratios do not capture the principles by which schooling is allocated if by these we mean principles related to the micro-social processes that generate unequal access to G, irrespective of overall access to G. The allocation mechanism referred to in the literature on the odds ratio cannot designate anything other than that which the odds ratio measures, a degree of intrinsic association, that is, the relative statistical chances of individuals from different categories accessing or not accessing a discrete good G.

In the present framework, to separate the effect of the micro-social processes generating unequal access to a good G from the opening up of access to G, that is, to measure inequality within a selection process, we will rely on the index developed by Bulle (2009) to this end and present this index in the next section.

Measuring Inequality Within a Selection Process: General Assumptions

The problem is as follows. We observe different situations of inequality relating to access to a discrete good G by comparing populations or one population at different time periods. Explaining these differences requires us to be able to separate the effects of the generative mechanisms underlying the observed inequalities, irrespective of the margins' role—that is, inequality within the selection process—from those which arise mechanically from the differences in the opening up of access to G in the various contexts under consideration. Note that the notion of selection process here covers all the effects of real-life selection processes, whether they are direct or indirect effects of cultural or economic factors, including the effects of voluntary choices that influence individuals' access to G, and so on. In fact, it covers the effects of all the processes access to G might depend on, the only thing it does not take into account is the proportion of individuals accessing to G, that is, the opening up of access to G.

The results of the selection process could be apprehended directly if we could class all the individuals of our population according to their relative chances of access to G—as if the results of the selection process could be represented by a queue, ranking individuals in decreasing order of their opportunity of access to G (by convention), their effective access then only depending on the opening up of access to G. We would need a formal representation of the results of such classification in order to measure the inequality of opportunity within the selection process. We could, for example, on the basis of this ranking, divide our population into n groups of an equal size ordered according to an increasing "distance" to G (if $n = 10$, we would have the 10 percent with the best opportunity of access, then the following 10 percent, for any n, we would have the first $\frac{100}{n}$ percent, the following $\frac{100}{n}$ percent, etc.). From that point, we would know the proportion of individuals belonging to a C_i category which is present in each small group constituted, and the proportion (complementary to one of the preceding proportion) of individuals who do not belong to C_i, that is, those who belong to $\overline{C_i}$, the complementary set of C_i in the population. Assuming that our population is sufficiently large and our constituted strata sufficiently narrow, we could characterize the distribution of the individuals belonging to C_i in our n small groups by a continuous curve (virtual limit of the histogram that has been set up) that would be defined on the interval [0, 1], just as the distribution of the individuals coming out of $\overline{C_i}$. The functions represented by the obtained curves are joint density functions $\tilde{f}(x, C_i)$. These functions depend both on the discrete variable that indicates that individuals belong to category C_i or $\overline{C_i}$ and on our continuous variable x defined on [0, 1], representing the distribution of relative opportunity of access to G—for example, $x = 0, 25$ indicates the cutting point in our ordered population separating the 25 percent of individuals with the greatest opportunity of access to G from the rest of the population.

Let us suppose that we have k C_i social categories in our population, allowing us to construct $k\tilde{f}(x, C_i)$ joint density functions. From these functions, we are able to calculate the proportion of individuals in our population belonging to the various social categories and present in each percentile group—for instance, in the top 10 percent, top 20 percent, and so on. This value can be calculated by an integral. On the basis of such joint density functions, for every $x_j \in [0, 1]$, expressing an opening up of access to the good G of $100x_j\%$, we would be able to calculate the proportion of individuals from each social category with access to G. Independent of the opening up of access to G, these functions characterize what has been defined as the results of the selection process.

Let us now consider the C_g^+ set of all the disadvantaged social groups (identified by a rate of access to G lower than the average rate of access x_j). Consequently, $\tilde{f}\left(x, C_g^+\right)$ tends to increase on [0, 1].[9] The continuous curve representing $\tilde{f}\left(x, C_g^+\right)$—a virtual limit of the histogram indicating the proportion of individuals from a disadvantaged social subgroup in each infinitesimal part of the population ordered according to relative opportunity of access to G—characterizes the overall outcomes of the selection process. The issue at stake is how to measure the inequality of these outcomes.

The problem is in fact simpler than it appears, because G is a discrete good. Its solution is not empirical but purely mathematical. We want to replace, from the data at hand, the dichotomous distribution of opportunity (putting in opposition access to *G* and exclusion from *G*) with a continuous distribution that expresses inequality of opportunity independently of the opening up of access to *G*. A multitude of continuous distributions are of course compatible with the data offered by the contingency table showing the rate of access to *G* by social group. The simplest, linear distribution—where $0 < \tilde{f}\left(x, C_g^+\right) < 1$, we will return to this condition subsequently—presents, like the straight line in statistics, particular advantages that here address our problem. By regularly spreading the discrepancies in the selection process results across the continuous distribution of relative opportunity of access to *G*, it allows comparisons across populations or across time. The slope of the straight-line segment characterizing this continuous distribution is, as we will see, a measure of inequality of opportunity within the selection process for access to *G*.

The measure of inequality within a selection process does not describe the resulting inequality, no matter how it is conceived, because the latter stems from the unequal outcomes of the selection process *and* the opening up of access to *G*. But it is an indispensable tool for comparing across contexts the relative effects these two factors have on observed inequality.

The essential elements underlying the meaning of the measure of inequality within a selection process for access to a discrete good *G* have been presented previously. We will devote the rest of this section to giving an account of its formal construction, its properties (especially its margin insensitivity), and its calculation.

The index proposed by Bulle (2009) is developed as follows.

Generally, the degree of inequality of access to a discrete good, *G*, can be ascribed to:

(1) Net results of the selection process in a broad sense—that is, the effects of all the factors influencing access to *G*—defined as a theoretical precedence ranking for access to *G*, or distance to *G*, and taking no account of actual access.
(2) Diffusion of *G* in society: the fraction of the overall population gaining access to *G*.

Inequality with respect to (1) is inequality within the selection process, defined as a measure permitting comparisons of the outcomes of microsocial processes affecting access to *G*, irrespective of the opening up of access to *G*.

Theoretical assumptions

It is assumed that access to a discrete good G has been derived from a latent continuous variable g which allows one theoretically to rank the whole population according to individual opportunities of access. By convention, a lower value of g will mean a greater opportunity of access to G. The variable g can be interpreted as a distance to G revealing the overall effect of the various factors in play in the process of access to G.

The population is of mass 1 and divided into k social subgroups: C_i is social subgroup i and $\overline{Ci}$ is the complementary subgroup of C_i within the population.

The k joint densities are $f(g, C_i)$ and the k joint cumulative distributions are $F(g, C_i)$ where C_i represents a nominal variable distinguishing individuals from C_i in the whole population.

It is assumed that the support of g is $\left[\underline{g}, \overline{g}\right]$, its density is $h(g)$ and its cumulative distribution is $H(g)$ (supposed to be strictly increasing so that its inverse function exists).

$x = H(g)$ is the fraction of the population whose "distance" to G is less than g ($g = H^{-1}(x)$ is the 100x-th percentile of the distribution of g). Thus, x is a continuous variable varying from 0 to 1.

The k joint cumulative distributions are defined as $\tilde{F}(x, C_i) = F(H^{-1}(x), C_i)$.

The k joint densities are defined as $\tilde{f}(x, C_i) = \frac{d}{dx}\tilde{F}(x, C_i)$.

On the basis of this formal framework, and from our knowledge of the dichotomous distribution of opportunity of access to G, we will construct k virtual joint densities, traced within a square of side 1, such that these joint densities allow us to compare opportunity within the selection process underlying access to G, irrespective of the overall proportion, x_j of individuals who attain G. To this end, we assume that these virtual joint densities are continuous and linear on the subsegment [0, 1], where these chances are not null and strictly below 100 percent. We then have two possible cases depending on whether the curve representing $\tilde{f}(x, C_i)$ intersects or not the base or the top of the square where $\tilde{f}(x, C_i)$ is traced.

It is assumed that the access of members of various C_i subgroups, given the overall proportion, x_j of individuals who attain G, has been derived from underlying continuous joint densities $\tilde{f}(x, C_i)$, such that the curves $\tilde{f}(x, C_i)$ are:

Case 1 (general case)—either straight-line segments so that $\tilde{f}(x, C_i) = \tilde{d}(x, C_i)$ on [0, 1] (see Figure 1).

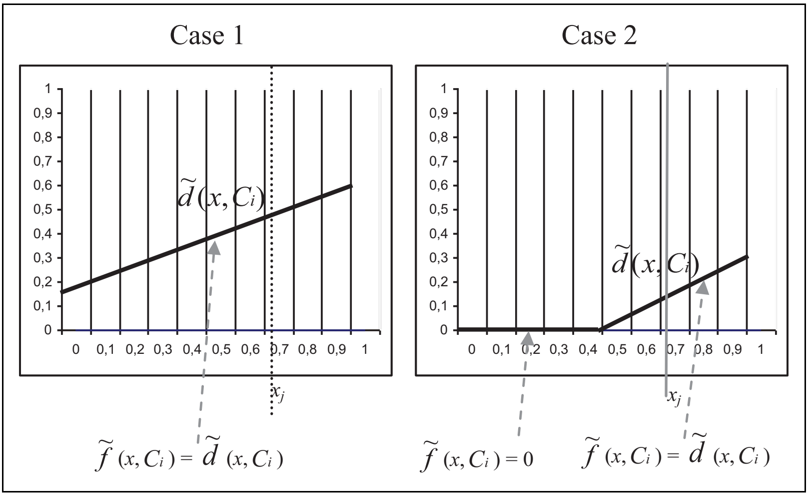

Figure 1. Virtual opportunity distributions.

Case 2—or broken line segments in cases where $\tilde{f}(x, C_i) = 0$ or $\tilde{f}(x, C_i) = 1$ on a segment of $[0, 1]$. Define $\tilde{d}(x, C_i)$, the straight-line segment such that $\tilde{f}(x, C_i) = \tilde{d}(x, C_i)$ where $0 < \tilde{f}(x, C_i) < 1$.

For each 2×2 contingency table showing the relationship between access to G and membership of a social subgroup C_i, there exists a virtual joint density $\tilde{f}(x, C_i)$ as defined previously, such that access of members of C_i to G could have been derived from $\tilde{f}(x, C_i)$; this can be shown graphically.

Define m_i the fraction of the whole population which belongs to C_i and r_i the rate of access to G of members of C_i. $\tilde{d}(x, C_i)$ is the straight-line segment characterizing $\tilde{f}(x, C_i)$ (case 1 or case 2 above) and $\tilde{a}_i$ the slope of $\tilde{d}(x, C_i)$. According to the definition of the joint density $\tilde{f}(x, C_i)$, in each interval, $[x_a, x_b] \subset [0, 1]$, $\tilde{f}(x, C_i)$ permits us to calculate the (virtual) fraction k_{ab} of members of C_i in the whole population belonging to this interval: $\int_{xa}^{xb} \tilde{f}(x, C_i)\, dx = k_{ab}$, so that we have $\int_0^1 \tilde{f}(x, C_i)\, dx = m_i$ and $\int_0^{xj} \tilde{f}(x, C_i)\, dx = r_i \times m_i$.

The family of joint densities $\tilde{f}(x, C_i)$ that fit $\tilde{f}(x, C_i)\ dx = m_i$ can be represented by Figure 2, for the case where the assumed straight-line segments $\tilde{d}(x, C_i)$ have a positive or zero slope (we observe graphically that this condition entails $r_i \leq x_j$, see Properties of the $(\tilde{a}_i)$ coefficients subsection 'c'). The complementary family of straight-line segments $\tilde{d}(x, C_i)$ with negative slope is symmetric to this family with respect to the axis $x = \frac{1}{2}$. As can be inferred from Figure 2, at any level x_j of diffusion of G, and for each possible distribution of access to G between C_i and $\overline{C_i}$ characterized by the access rate r_i to G of individuals from C_i, there exists a virtual joint density $\tilde{f}(x, C_i)$ belonging to case 1 or 2 above which can be associated with this distribution.

Properties of the $(\tilde{a}_i)$ coefficients

As showed in Bulle (2009:3.2), several general properties of the $(\tilde{a}_i)$ coefficients can be deduced from the previous assumptions, some complementary graphical justifications are offered here.

(a) We have $\tilde{f}(x, C_i) + \tilde{f}(x, \bar{C}_i) = 1$, which is an obvious consequence of the definition of the joint densities $\tilde{f}(x, C_i)$ and $\tilde{f}(x, \bar{C}_i)$. We can then deduce (from Figure 2 for instance) that if the slope of $\tilde{d}(x, C_i)$ is $\tilde{a}_i$, the slope of $\tilde{d}(x, \bar{C}_i)$ is $(-\tilde{a}_i)$.

(b) When there is no distinction in terms of relative opportunity of access to G between C_i and the whole population, the representative curve of $\tilde{f}(x, C_i)$ is the straight-line segment whose equation is $\tilde{d}(x, C_i) = m_i$, so that $\tilde{a}_i = 0$, so that we find the same proportion of individuals from C_i in each percentile group than in the whole population.

(c) $\tilde{a}_i < 0$ (>0) iif for any x_j defining the overall rate of access to G, the access rate of members of C_i is higher (lower) than overall rate of access to G, x_j.

The surface $x_j \times m_i$ is represented by the square at the bottom left of Figure 2. It is then obvious that:

$$\int_0^{xj} \tilde{f}(x, C_i)\ dx = r_i \times m_i < x_j \times m_i \Leftrightarrow \tilde{a}_i > 0$$

(d) The $(\tilde{a}_i)$ coefficients are insensitive to marginal distributions—that is, whatever the values of the margins x_j and m_i, they do not restrict the degree of inequality measured by $\tilde{a}_i$.

Insensitivity to x_j variations stems from the definition of $\tilde{a}_i$ in a reference frame independent of variations in overall access to G, x_j. Furthermore, for any value of m_i, the family of joint densities

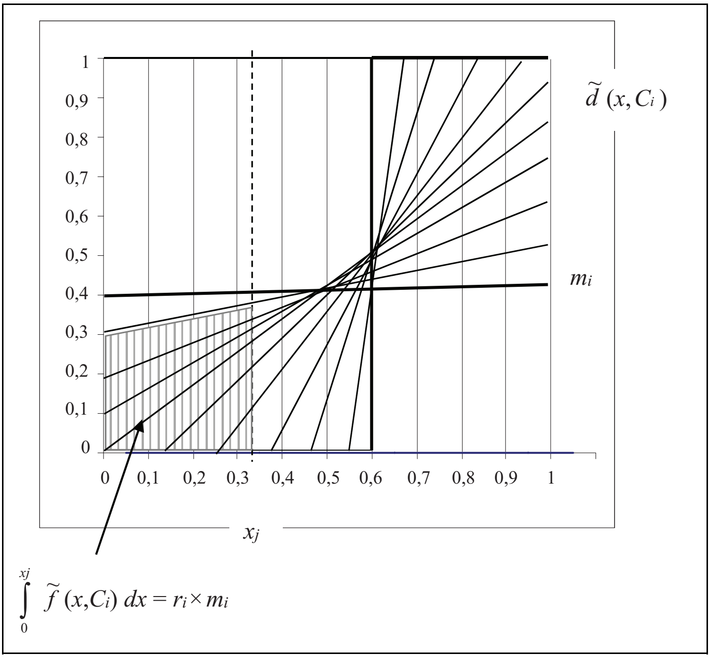

Figure 2. Family of virtual joint densities $\tilde{f}(x, C_i)$.

$\tilde{f}(x, C_i)$ that fit $\int_0^1 \tilde{f}(x, C_i)\,dx = m_i$ is represented in Figure 2, showing that the $\tilde{a}_i$ coefficient, slope of $\tilde{d}(x, C_i)$—case 1 or case 2 in the Theoretical assumptions subsection—can take any value in its interval of variation $[-\infty, +\infty]$. Therefore, the value of $\tilde{a}_i$ is not constrained by the margins values.

Several specific properties of the $(\tilde{a}_i)$ coefficients can be shown for the general case (case 1 in the Theoretical assumptions subsection)—that is, when none of the $\tilde{d}(x, C_i)$ straight-line segments intersects the top or the bottom of the square where the $\tilde{f}(x, C_i)$ curves are inscribed.

(e) $|\tilde{a}_i| \leq 1$ (evident graphically).

(f) If subgroups are aggregated, $C_k = \cup C_i$, the slope of $\tilde{d}(x,\ C_k)$ is equal to the sum of the slopes of $\tilde{d}(x,\ C_i)$: $\tilde{a}_k = \Sigma \tilde{a}_i$.

(g) If $C_k = \cup C_i$ represents the whole population, then $\sum_{i=1}^{k} \tilde{a}_i = 0$. That is evident from (f).

Interpretation of $\tilde{a}_g$ as an overall measure of inequality within the selection process

$\cup C_i = C_g^+ \left(C_g^-\right)$ is the set of subgroups C_i whose members have lower (higher) than average access to G.

According to the Theoretical assumptions and Properties of the $(\tilde{a}_i)$ coefficients subsections:

(1) The coefficient $\tilde{a}_g$ opposes the opportunity of access to a discrete good G of disadvantaged social groups $\left(C_g^+\right)$ to that of advantaged social groups $\left(C_g^-\right)$ within the whole population: $\tilde{a}_g\,(-\tilde{a}_g)$ represents the slope of the straight-line segment $\tilde{d}\left(x, C_g^+\right)\,\left[\tilde{d}\left(x, C_g^-\right)\right]$ which characterizes the inequality of the continuous distribution of relative opportunity of individuals from $C_g^+\left(C_g^-\right)$, irrespective of access to G—that is, the selection process results.[10]

(2) The coefficient $\tilde{a}_g$ is insensitive to margins (x_j, the overall access rate to G, and m_g, the fraction of the whole population in C_g^+). Consequently, the coefficient $\tilde{a}_g$ is a margin-insensitive index of inequality within the selection process.

Calculation of $\tilde{a}_g$

Table 4 shows the relationship between access to G and membership of C_g^+, given x_j, the overall proportion of individuals who gain access to G; m_g, the fraction of the population that belongs to C_g^+; and r_g, the rate of access to G for members of C_g^+.

$\tilde{d}\left(x, C_g^+\right)$ is the straight-line segment characterizing the joint density function $\tilde{f}\left(x, C_g^+\right)$ defined in case 1 and case 2 in the Theoretical assumptions subsection, and $\tilde{a}_g$ its slope.

According to our assumptions, $\tilde{f}\left(x, C_g^+\right) = \tilde{d}\left(x, C_g^+\right)$, where $0 < \tilde{f}\left(x, C_g^+\right) < 1$; as seen in the Theoretical assumptions subsection, we have $\int_0^1 \tilde{f}\left(x, C_g^+\right) dx = m_g$ and $\int_0^{xj} \tilde{f}\left(x, C_g^+\right) dx = r_g \times m_g$.

Define the straight line $d\left(x, C_g^+\right)$ and its slope a_g such that

Table 4. Parameters of the 2 × 2 Contingency Table (I).

	Access to *G*	No access to *G*	
C_g^-	α	β	$\alpha + \beta = 1 - m_g$
C_g^+	γ	δ	$\gamma + \delta = m_g$
	$\alpha + \gamma = x_j$	$\beta + \delta = 1 - x_j$	1

$$\int_0^1 d\left(x, C_g^+\right) dx = m_g \text{ and } \int_0^{xj} d\left(x, C_g^+\right) dx = r_g \times m_g.$$

It can easily be shown that $d\left(x, C_g^+\right)$ passes through two points A and B, situated in the middle of the two horizontal segments in Figure 3 and that $d\left(x, C_g^+\right)$ is the regression line between the two binary variables concerned (access to G and membership of C_g^+); a_g is thus a regression coefficient; moreover, $\frac{a_g}{2}$ is the association coefficient obtained by subtracting the proportion of members of C_g^+ in the selected group from that in the nonselected group.

From the definition of $d\left(x, C_g^+\right)$ mentioned previously, we deduce that the equation of $d\left(x, C_g^+\right)$ is:

$$y = a_g x + m_g - \frac{a_g}{2} \text{ and that } a_g = \frac{2 \times m_g \times \left(x_j - r_g\right)}{\left(1 - x_j\right) \times x_j}.$$

From these equations, we can also deduce that $d\left(x, C_g^+\right)$ intersects the two sides of the square shown in Figures 1 to 3 iif a_g satisfies $\frac{a_g}{2} \leq m_g \leq 1 - \frac{a_g}{2}$. (I).

This gives us the two cases presented in the Theoretical assumptions subsection.

(1) Condition (I) is fulfilled: $\frac{a_g}{2} \leq m_g \leq 1 - \frac{a_g}{2}$, such that $d\left(x, C_g^+\right)$ intersects the two sides of the square where the $\tilde{f}\left(x, C_g^+\right)$ curve is inscribed. We deduce that the conditions defining $\tilde{d}\left(x, C_g^+\right)$ and $d\left(x, C_g^+\right)$ are the same on [0, 1]. We are thus in the general case: $\tilde{f}\left(x, C_g^+\right) = \tilde{d}\left(x, C_g^+\right) = d\left(x, C_g^+\right)$ on [0, 1], and therefore $\tilde{a}_g = a_g$.

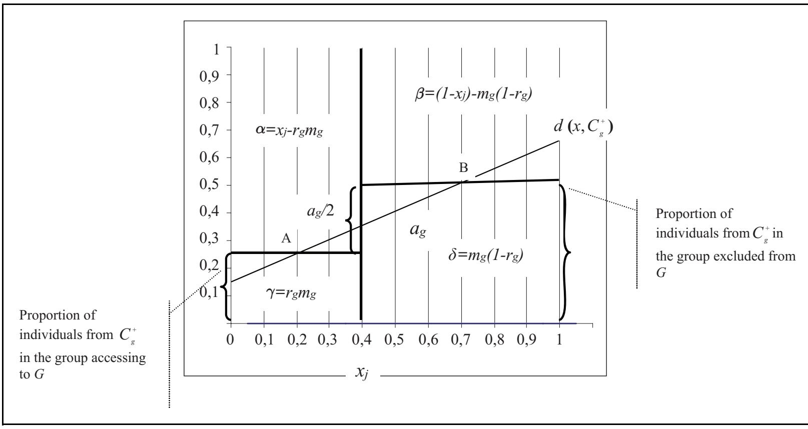

Figure 3. Graphic representation of the contingency table data presented on Table 4 and regression line $d\left(x, C_g^+\right)$.

Then—and essentially whenever the general case applies—$\tilde{a}_g$ corresponds to twice the value of the difference in proportions $\frac{a_g}{2}$ when comparing the contingency table columns.

(2) In other cases, condition (I) is not fulfilled. The function $d\left(x, C_g^+\right)$ therefore takes negative values or values above 1 on [0, 1]. The curve representing $\tilde{f}\left(x, C_g^+\right)$ is thus a broken line segment on [0, 1]—that is, $\tilde{f}\left(x, C_g^+\right) = 0$ or $\tilde{f}\left(x, C_g^+\right) = 1$ on a segment of [0, 1]. In the opposite case, $\tilde{f}\left(x, C_g^+\right) = \tilde{d}\left(x, C_g^+\right) = d\left(x, C_g^+\right)$ on [0, 1], which would contradict the hypothesis that $d\left(x, C_g^+\right)$ intersects the top or base of the square where the $\tilde{f}\left(x, C_g^+\right)$ curve is inscribed. Therefore, $\tilde{d}\left(x, C_g^+\right) \neq d\left(x, C_g^+\right)$ and $\tilde{a}_g \neq a_g$.

Figure 4 illustrates case (2) when $d\left(x, C_g^+\right)$ intersects the base of the square where $\tilde{f}\left(x, C_g^+\right)$ is traced.

Define x_k as the abscissa of the intersection between $\tilde{d}\left(x, C_g^+\right)$ and the x-abscissa (Figure 4).

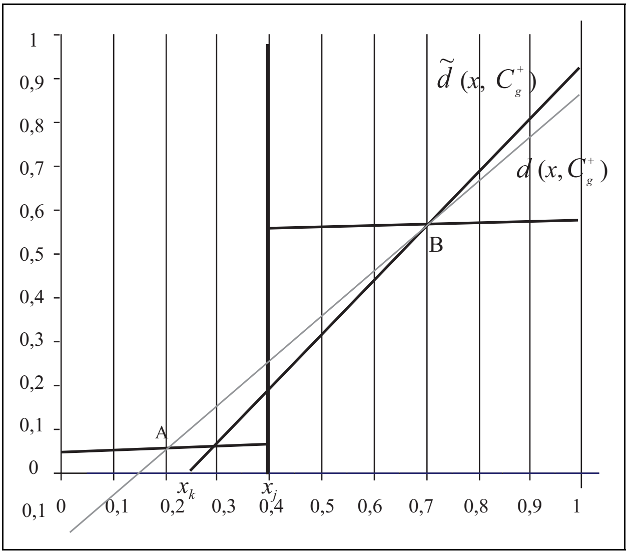

Figure 4. A specific case.

$\tilde{d}\left(x,\ C_g^+\right)$ and $d\left(x,\ C_g^+\right)$ are, respectively, the solutions of:

$$\int_{xk}^{1} \tilde{d}\left(x,\ C_g^+\right) dx = m_g \text{ and } \int_{xk}^{xj} \tilde{d}\left(x, C_g^+\right) dx = r_g \times m_g$$

$$\int_{0}^{1} d\left(x,\ C_g^+\right) dx = m_g \text{ and } \int_{0}^{xj} d\left(x,\ C_g^+\right) dx = r_g \times m_g.$$

When $d\left(x,\ C_g^+\right)$ intersects the top or base of the square where $\tilde{f}\left(x,\ C_g^+\right)$ is traced—that is, when condition (I) does not apply—the calculation of $\tilde{a}_g$ is specific and is provided in the Appendix to this article. Note that there is no discontinuity in the calculation of $\tilde{a}_g$ in borderline cases when one formula ceases to apply and another formula begins to be applicable. In extreme cases where none of the members of C_g^+ gain access to G or where none of the members of C_g^- fail to gain access to G, $\tilde{a}_g$ can only be approximated.

Finally, even if, in the general case—condition (I) fulfilled—the inequality index $\tilde{a}_g$ is calculated in the same way as a conventional measure of association $\left(\frac{a_g}{2}\right)$, its meaning is different; and while $\frac{a_g}{2}$, which can reach a maximum of 1 only if the nonselected fraction of the population equals the fraction belonging to C_g^+, is sensitive to the relative size of the social groups, the index $\tilde{a}_g$ is not, so the values of these two indices coincide only in the general case, as shown previously.

The selection process results, inequality within the selection process, and the issue of linearity

It should be noted that the assumption of linearity that underlies the measurement of the inequality within the selection process $\tilde{a}_g$ is a simple heuristic because G is a discrete good. We have no need to know the form taken in reality by the (continuous) distribution of opportunity underlying access to a good G—that is, the selection process results irrespective of overall access to G. Knowing such a distribution would allow us to resolve another problem regarding a continuous good G.

For example, if G represented a certain salary level S, and inequality between men and women was the subject of the analysis, the ranking of the population in terms of salaries would not be useful for determining inequality of opportunity within the selection process for access to the defined salary level S. The study of access to salary level S would just lead to the application of a cutting point in this ranking. By regularly spreading the discrepancies in the selection process results across the continuous distribution of relative opportunity of access to this salary level S, linearity allows us to compare the unequal outcomes of the selection process between populations or across time.

Now, if G was a continuous good, the comparison of inequality within the selection process between populations, or between periods of time, would require to compare inequality within the selection process for access to the various percentile groups at stake. Alternatively, let us suppose that there is a hierarchical ranking of n discrete goods G_k (constituting a vertical ordering of, for instance, education levels), such that all individuals accessing any higher ranked good G_{k+1} would have access to the lower ranked good G_k. We would dispose in such a case of n overall measures of inequality in the selection process for access to the n goods G_k. We can consider that these n ranked goods, from the less selective to the most selective good, $G_{k,}$ are cutting points within the distribution of relative opportunity of access to a continuous good G (if the goods G_k are levels of education, then G represents the "formal education" good). These various cutting points may suffice to assess, by linear extrapolations for instance, inequality within the selection process for access to each specific percentile group (Bulle 2009:581).

Let us take the example of a specific case to illustrate the role of the assumption of linearity. We assume we have a selection process for access to a discrete good G based on a specific rule of selection such that the equation of the curve representing the (real) joint density function $\tilde{f}\left(x,\ C_g^+\right)$ characterizing the selection process results is $y = x^2$, which increases from $y = 0$ to $y = 1$ on the interval [0, 1], the shape of the curve being that of a parabola. The area under the curve on the interval [0, 1] is 1/3, so that the fraction of the whole population in C_g^+ is 1/3. We may calculate the successive values of inequality within the selection process for access to various percentile groups. For instance, we calculate the $\tilde{a}_{gk}$ coefficients assuming the linearity of the virtual joint density functions $\tilde{f}_k\left(x,\ C_{gk}^+\right)$ (where these functions are not null and inferior to 1), with G_k representing specific percentile groups: the top k percent. We find the following values for $\tilde{a}_{gk}$, k varying by 10 from 10 to 90: 0.77, 0.86, 0.95, 1.03, 1.11, 1.19, 1.27, 1.35, and 1.42. These values are based on case 2 in the Appendix, because, in each case $\tilde{d}_k\left(x,\ C_{gk}^+\right)$ intersects the base and one side of the square where $\tilde{f}_k\left(x,\ C_{gk}^+\right)$ is traced. Then, we can conclude that inequality within the selection process for access to G would increase, somewhat linearly, from an opening up of access to G of 10% to an opening up of 90%. The linearity assumption underlying the calculation of inequality allows us to compare the selection process results for access to G between such contexts defined by differing overall levels of access to G.

Note that if we calculate the values of the various odds ratios in order to compare relative opportunity of access to G between these same contexts, we find successively 195.2, 51.5, 27.9, 19.3, 15.4, 13.9, 13.9, 16.0, and 25.2. Relative opportunity of access to G, as measured by the odds ratio, decreases rapidly from an opening up of G of 10% to an opening up of 60% and then increases somewhat to an opening up of 90%. The diagnostic of the evolution of opportunity is thus quite different from that offered by the measure of inequality within the selection process, and such a difference does express the various aspects of inequality these indexes capture, that is, one compares the outcomes of the micro-social processes that generate unequal access to G across contexts, irrespective of the opening up of access to G, and the other one compares a degree of intrinsic association between access to G and social stratification.

Comparison of the $\tilde{a}_g$ Index With Classical Inequality of Opportunity Measures

In the following, we suppose that $a_g = \frac{2\times m_g \times \left(x_j - r_g\right)}{\left(1-x_j\right)\times x_j}$ satisfies condition (I): $\frac{a_g}{2} \leq m_g \leq 1 - \frac{a_g}{2}$, that is, the general case (case 1 in the Theoretical assumptions subsection) applies, so that $\tilde{d}\left(x,\ C_g^+\right) = d\left(x,\ C_g^+\right)$ and, thus, $\tilde{a}_g = a_g$; as stated in the Calculation of $\tilde{a}_g$ subsection, the equation of $d\left(x,\ C_g^+\right)$ is $y = a_g\ x + m_g - \frac{a_g}{2}$.

Relationship With the Ratio of Proportions

The access rate for members of C_g^+ is $\boldsymbol{r}_g^+ = \frac{\gamma}{m_g}$

$$\boldsymbol{r}_g^+ = \left[\frac{a_g}{2m_g} \times x_j^2\right] + \left[\left(1 - \frac{a_g}{2m_g}\right) \times x_j\right]$$

$$\partial\, \boldsymbol{r}_g^+ / \partial x_j = \left[\frac{a_g}{m_g} \times x_j\right] + \left[1 - \frac{a_g}{2m_g}\right]$$

$$\partial\, \boldsymbol{r}_g^+ / \partial x_j^2 = \frac{a_g}{m_g}.$$

The derivative of $\boldsymbol{r}_g^+$ at x_j is, according to the conditions met by the parameters, positive ($a_g \geq 0$ and $m_g \geq \frac{a_g}{2}$ since the general case applies). This is a consistent result: If inequality within the selection process $\tilde{a}_g$ is held constant, the rate of access to G $\boldsymbol{r}_g^+$ increases with diffusion of G. This rate increases with increasing speed as $a_g > 0$.

Define $R_g = \boldsymbol{r}_g^- / \boldsymbol{r}_g^+ = \frac{mg}{1-mg} \times \frac{\alpha}{\gamma}$ as the ratio of proportions.

Note that R_g is not margin insensitive. For instance, if x_j, the fraction of the population gaining access to G, increases, giving $x_j' = \lambda\ x_j$ with $\lambda > 1$, inequality R_g can remain stable only if $\alpha' = \lambda\alpha$ and $\gamma' = \lambda\gamma$ with $\lambda < \frac{1-m_g}{\alpha}$.

Substituting α with $x - \gamma$ and γ with $(½\ a_g \times x_j^2) + (m_g - ½\ a_g)\ x_j$ (see Tables 4 and 5) in the formulae for R_g mentioned previously, simplifying by x (which is not null) and factorizing, we obtain:

Table 5. Parameters of the 2 × 2 Contingency Table (II).

	Access to G	No access to G	
C_g^-	$x-\gamma$	$1-m_g-x_j+\gamma$	$1-m_g$
C_g^+	γ[a]	$m_g-\gamma$	m_g
	x_j	$1-x_j$	1

[a] $\gamma = (½\ a_g \times x_j^2) + (m_g - ½\ a_g)\ x_j$.

$$R_g = \frac{m_g}{1-m_g} \times \frac{a_g(1-x_j)+2(1-m_g)}{-a_g(1-x_j)+2m_g}\ \textit{and}$$
$$\partial R_g/\partial x_j = 2\frac{m_g}{1-m_g} \times \frac{-a_g}{(a_g x_j + 2m_g - a_g)^2} \leq 0.$$

$\boldsymbol{r}_g^- / \boldsymbol{r}_g^+$ decreases with diffusion of G, inequality within the selection process $\tilde{a}_g$ being held constant. Note that this contributes to explain the overall decrease in inequality of opportunity observed on the basis of such an index (see Note 4), as even if inequality within the selection process remained unchanged, such a decrease would be observed due to the expansion of educational systems.

Define the ratio of exclusion rates:

$$\bar{R}_g = \frac{1-m_g}{m_g} \times \frac{\delta}{\beta} = \frac{1-m_g}{m_g} \times \frac{a_g x_j + 2m_g}{-a_g x_j + 2(1-m_g)}.$$

Like R_g, $\bar{R}_g$ is constrained by marginal values. For instance, if the fraction $1-x_j$ of the population without access to G increases, $(1-x_j)' = \lambda(1-x_j)$ with $\lambda > 1$, inequality $\bar{R}_g$ can remain stable only if $\delta' = \lambda\delta$ and $\beta = \lambda\beta$ with $\lambda < \frac{mg}{\delta}$.

$$\partial \bar{R}_g/\partial x_j = \frac{1-m_g}{m_g} \times \frac{2a_g}{[-a_g x_j + 2(1-m_g)]^2} \geq 0$$

Relative risk of exclusion from G for members of C_g^+ increases with diffusion of G. Measures of inequality R_g and $\bar{R}_g$ vary in an inconsistent way with respect to the complementary aspect of the inequality they measure. Thus, they systematically sustain contradictory accounts of evolution of inequality when inequality within the selection process is held constant.

Relationship With the Odds Ratio

The odds ratio Θ_j compares relative chance of access to G and relative risk of exclusion from G for members of C_g^- and C_g^+. It takes into account both selected (gain access to G) and nonselected (fail to gain access to G) individuals and is not sensitive to marginal variations.

$$\Theta_j = R_g \times \bar{R}_g = \frac{\alpha/\beta}{\gamma/\delta}$$

We obtain $\Theta_j = \frac{a_g(1-x_j)+2(1-m_g)}{-a_g(1-x_j)+2m_g} \times \frac{a_g x_j + 2m_g}{-a_g x_j + 2(1-m_g)}$

Θ_j can be written $\Theta_j = \frac{u(x_j)}{u(x_j)-2a_g}$

With $u(x_j) = -a_g^2 x_j^2 + \left(a_g^2 - 4a_g m_g + 2a_g\right)x_j + 2a_g m_g + 4m_g - 4m_g^2$

Note that, according to the conditions met by the parameters, it can be shown that Θ_j varies positively with a_g.

$$\partial\Theta_j/\partial x_j = \frac{-2a_g \times u'(x_j)}{\left(u(x_j) - 2a_g\right)^2} = \frac{4a_g^3\left[x_j - \left(1/2 - 2m_g/a_g + 1/a_g\right)\right]}{\left(u(x_j) - 2a_g\right)^2}.$$

That is, $A = \frac{1}{2} - 2\frac{m_g}{a_g} + \frac{1}{a_g}$, as $a_g \geq 0$ the sign of $\partial\Theta_j/\partial x_j$ is thus the sign of $(x_j - A)$

If, $A \leq 0$, $\forall x_j \in [0, 1]$, $(x_j - \mathrm{A}) \geq 0$: Θ_j increases from $x_j = 0$ to $x_j = 1$.

If $0 < A < 1$, $(x_j - A) \geq 0 \Leftrightarrow x_j \geq A$: Θ_j decreases from $x_j = 0$ to $x_j = A$ and increases until $x_j = 1$.

Note that if $m_g = ½$ the curve $\Theta_j(x_j) = \left[\frac{a_g x_j(1-x_j)+a_g+1}{a_g x_j(1-x_j)-a_g+1}\right]$ is symmetrical to the axis $x_j = ½$.

If $A \geq 1, \forall x_j \in [0, 1], (x_j - A) \leq 0$: Θ_j decreases from $x_j = 0$ to $x_j = 1$.

Relationship With the Difference in Access Rates

The difference in access rates, D_g is calculated by subtracting the proportion of the disadvantaged subgroup which is selected from the proportion of the advantaged subgroup which is selected.

$D_g = \boldsymbol{r}_g^- - \boldsymbol{r}_g^+$ so that $D_g = \frac{\alpha}{1-m_g} - \frac{\gamma}{m_g}$.

D_g can reach its maximum of 1 only if the proportion of members of C_g^- in the population equals the proportion x_j of individuals selected. Therefore, it is not margin insensitive.

$$D_g = \frac{a_g}{2} \times \frac{x_j(1-x_j)}{m_g(1-m_g)}.$$

Inequality D_g increases linearly with a_g.

$$\partial D_g/\partial x_j = \left(x_j - {}^1\!/\!_2\right) \times \frac{-a_g}{m_g(1-m_g)}.$$

The difference in access rates increases until $x_j = ½$ and then decreases. D_g tends toward 0 when x_j tends toward 0 or toward 1, that is, when access to G is either rare or widespread. The axis of symmetry $x_j = ½$, with regard to variations of D_g, is explained by the symmetry of the parts played by the selected and the nonselected individuals.

Relationship With the Phi Coefficient of Association

$\varphi^2 = \frac{\chi^2}{N}$.

N indicates the size of the population.

In the case of a 2 × 2 table, the phi coefficient can be considered as a coefficient of linear correlation between two binary variables.

$\varphi = \frac{(\alpha \times \delta - \beta \times \gamma)}{\sqrt{(\alpha+\beta) \times (\gamma+\delta) \times (\alpha+\gamma) \times (\beta+\delta)}}$.

Define σ_g and σ_j as the standard deviations of the two variables (membership of C_g^+ and access to G). $\varphi_j = D_g \times \frac{\sigma_g}{\sigma_j}$ and $\varphi_j = \frac{a_g}{2} \times \frac{\sigma_j}{\sigma_g}$. Interpreted as a correlation coefficient, φ_j is the geometric mean of the two regression coefficients D_g and $\frac{a_g}{2}$. We have seen that D_g cannot express the degree of inequality regardless of marginal variations. The same is true for $\frac{a_g}{2}$ (see Calculation of $\tilde{a}_g$ subsection). This is also true for φ_j.

$$\varphi_j = \frac{a_g}{2} \times \frac{\sqrt{x_j(1-x_j)}}{\sqrt{m_g(1-m_g)}}.$$

Inequality φ_j increases linearly with a_g.

$$\partial\varphi_j/\partial x_j = \frac{a_g(1-2x_j)}{4\sqrt{x_j(1-x_j) \times m_g(1-m_g)}}.$$

φ_j increases with diffusion of G until $x_j = ½$ and then decreases. φ_j tends toward 0 when x_j tends toward 0 or when x_j tends toward 1. As it is the case

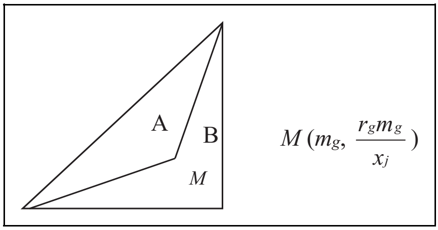

Figure 5. Lorenz concentration curve.

for D_g, the axis of symmetry $x_j = ½$, with respect to variations of φ_j, is explained by the symmetry of the part played by the selected and the nonselected individuals.

Relationship With the Gini Coefficient

The Gini coefficient, G_j, measures the concentration of access to G in the present framework. It represents a ratio of areas on the Lorenz concentration curve diagram ($G_j = \frac{A}{A+B}$ in Figure 5). The Lorenz curve is obtained by plotting cumulative shares of G (originally income shares) against cumulative percentage of the population. Note that we consider the aggregated subgroup C_g^+ here, in order to calculate a general relationship among G_j, a_g, x_j, and r_g, so that the Lorenz curve is characterized by only one point M.

We find $G_j = m_g - \frac{r_g m_g}{x_j}$: the fraction of the population that belongs to C_g^+ minus the proportion of members of C_g^+ in the selected group (or the proportion of members of C_g^- in the selected group minus the fraction of the population that belongs to C_g^-).

Thus, we have $G_j = \frac{a_g}{2} \times \left(1 - x_j\right)$ with $a_g = \frac{2 \times m_g \times \left(x_j - r_g\right)}{\left(1 - x_j\right) \times x_j}$ (general case).

Inequality G_j increases linearly with a_g.

$$\partial G_j / \partial x_j = -\frac{a_g}{2}.$$

The Gini coefficient, G_j, decreases linearly with diffusion of G across the population, with a speed proportional to a_g. From this concentration defined by individuals gaining access to G, the concentration defined by individuals who fail to gain access to G can be deduced:

$$\bar{G}_j = \frac{\delta}{1 - x_j} - m_g = \frac{a_g}{2} x_j.$$

These two concentrations vary in opposite directions when diffusion of G increases. The expression of these two concentrations with respect to coefficient $\tilde{a}_g(= a_g$, general case) and diffusion rate x_j clarifies the paradox according to which these indices simultaneously indicate very different degrees of inequality. This occurs when $x_j \ll \frac{1}{2}$ or $x_j \gg \frac{1}{2}$. This is intuitively sensible: The concentrations of selected and nonselected individuals measured by the Gini coefficient show, for a definite value of inequality within the selection process a_g, degrees of inequality that are more divergent when access to G is either very concentrated or distributed throughout the population.

Relationship with the Marginal Matching Coefficient

Blackburn and Marsh (1991), and Blackburn et al. (1995) proposed a measure—the marginal matching coefficient MM—to address the problem of sensitivity to marginal variations in segregation analysis. MM requires one to provide matched distributions in the two margins of the basic segregation table, that is, to change the definition of advantaged versus disadvantaged units or, as here, social subgroups and define two sets of units such that their distribution in each period is identical to the distribution of the segregation criteria (fractions of men and women in the labor force for instance; or as here, fraction of individuals gaining access to G) such that it is possible in principle for every member of the advantaged subgroups, and none of the members of the other subgroups, to meet the criteria for selection. This calculation is done by ranking the units (social subgroups in the present example) according to the fraction without access to G—this ranking may be carried out finely using an index based on a continuous scale of social advantage—and then by calculating the cumulative fraction of the population starting at the top of this ranking, and moving along the cumulative distribution of the social subgroups, until it equals $1 - x_j$ (the proportion of the population excluded from G). This procedure "matches" marginal totals $1 - x_j$ and x_j to the respective proportions of disadvantaged and advantaged subgroups m_{MM} and $1 - m_{\mathrm{MM}}$ in the population. Therefore, the disadvantaged group C^{+}_{MM} contains the same number of individuals, as there are individuals excluded from access to G, while the advantaged subgroup C^{-}_{MM} contains the same number of individuals as there are individuals accessing G. Marginal matching ensures that exclusion from G of members from the disadvantaged subgroups has a stable meaning with regard to segregation, as it is always formally possible to observe complete segregation in the population. Thus, MM is insensitive to marginal variations.

On the basis of this segregation table with matched margins, several statistics of association coincide with MM, in particular, the difference in access rates, the difference in proportions when comparing contingency table columns, and the phi coefficient: $D_{\mathrm{MM}} = \frac{a_{\mathrm{MM}}}{2} = \varphi_{\mathrm{MM}} = \mathrm{MM}$.

Therefore, as in the general case (I), we have $\tilde{a}_{\mathrm{MM}} = a_{\mathrm{MM}}$; MM is half the value of the inequality within the selection process $\tilde{a}_{\mathrm{MM}}$, where $\tilde{a}_{\mathrm{MM}}$ is defined on the basis of the fraction $1 - m_{\mathrm{MM}}$ of the advantaged group matched with the fraction x_j of individuals accessing to G. Nevertheless, there is no general formal link between MM and $\tilde{a}_g$. C^+_{MM} is constructed from the aggregation of the least advantaged subgroups until the fraction $m_{\mathrm{MM}} = 1 - x_j$. When the general case (I) holds for each of the subgroups of interest, according to property (f) in Properties of the $(\tilde{a}_i)$ coefficients subsection, the inequality coefficient $\tilde{a}_{\mathrm{MM}}$ associated with C^+_{MM} is the sum of the inequality coefficients associated with the aggregated subgroups; we also have: $\sum_{\tilde{a}_i>0} \tilde{a}_i = \sum_{\tilde{a}_i<0} (-\tilde{a}_i) = \tilde{a}_g\left(= a_g\right)$. Thus, we can say that, in this case, $\mathrm{MM} = \frac{a_{\mathrm{MM}}}{2}$ is less than or equal to $\frac{\tilde{a}_g}{2}$, since $\tilde{a}_g$ is a maximum; when $m_g = 1 - x_j$, we have $\mathrm{MM} = \frac{\tilde{a}_g}{2}$.

Index Variations With G Diffusion, Some Illustrations

Figures 6 to 8 illustrate the variation of the classical indices listed previously with respect to diffusion of G (from $x_j = 10\%$ to $x_j = 90\%$) when the degree of inequality within the selection process $\tilde{a}_g$ and fraction m_g of the population belonging to C^+_g are held constant.

As defined, D_g and φ_j are positive and thus range from 0 to 1, which is the variation interval of G_j, with 0 indicating no relationship. To make comparisons of how these indices vary with diffusion of G easier, the variation intervals of indices R_g, $\bar{R}_g$ and Θ_j have been standardized with the following measures: $P_g = \frac{R_g - 1}{R_g + 1}$; $S_g = \frac{\bar{R}_g - 1}{\bar{R}_g + 1}$; $Q_j = \frac{\theta_j - 1}{\theta_j + 1}$. As defined, R_g, $\bar{R}_g$, and Θ_j range from 1 to ∞ and thus P_g, S_g, and Q_j range from 0 to + 1, with 0 indicating no relationship.

Measuring Inequality Within the Selection Process: Return to the 1944 Act Example

We now have available all the instruments and data required to assess the evolution of inequality within the selection process for access to lengthy secondary education for English children born in the first half of the

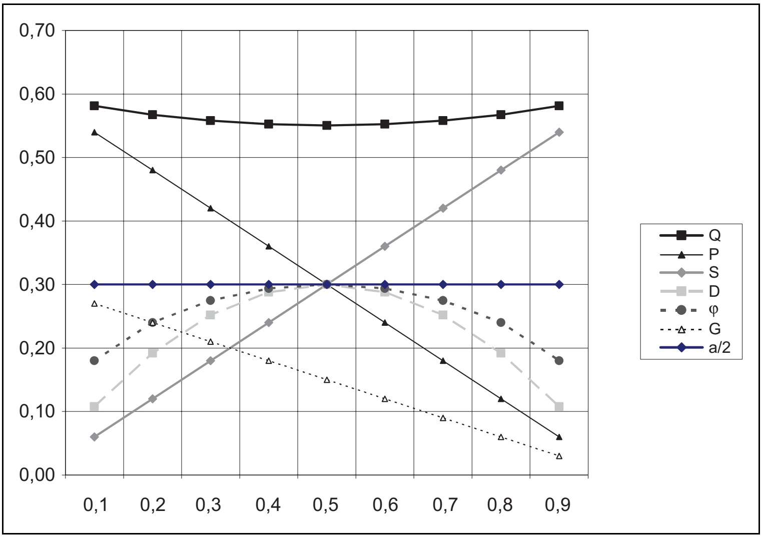

Figure 6. Variation of inequality indices with respect to G's diffusion–$m_i = 0.5$ and $a_i = 0.6$.

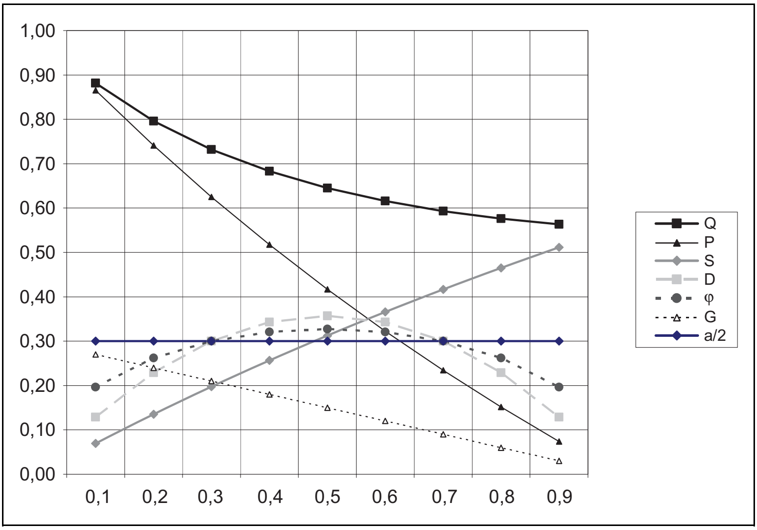

Figure 7. Variation of inequality indices with respect to G's diffusion–$m_i = 0.3$ and $a_i = 0.6$.

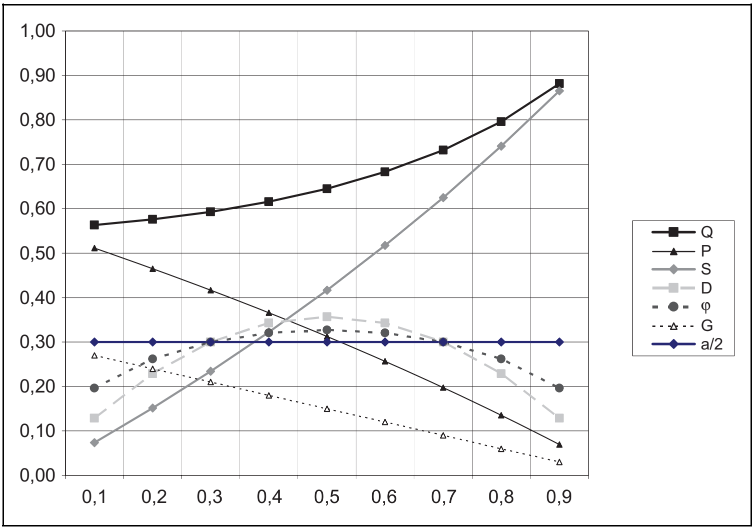

Figure 8. Variation of inequality indices with respect to *G*'s diffusion–$m_i = 0.7$ and $a_i = 0.6$.

twentieth century. To begin with, we look at the results obtained from data studied by Little and Westergaard. We have supplemented the data provided in their tables with data from Floud (1954) and the Crowther Report (1959). Our discussion focuses on boys in order to be able to extend this study with an analysis of data published by Halsey et al. (1980) and also used by Blackburn and Marsh (1991).

We have seen that Little and Westergaard's account of inequalities in access to selective secondary education was mixed, depending on whether it was based on the evolution of chance of access or risk of exclusion. The odds ratio, which takes both into account, compares the chances of children of parents belonging to the more advantaged categories—Professionals and Managers—with the chances of children of parents belonging to the less advantaged categories—Semiskilled and Unskilled Manual Workers—and has a value of 58 for the cohort born before 1910, 22, 15, and 16, respectively, for the three following cohorts (data from Table 1). Based on this index, one might conclude that initially there was a significant decrease in inequality of opportunity, with no reduction over the last two cohorts. When the same statistic, the odds ratio, is calculated for boys only (data from Table 6), the decrease in inequality is less pronounced, with a slight strengthening over the last two cohorts the—values for successive cohorts are 25, 18, 13, and 16, respectively. However, if inequality within the selection process is measured by $\tilde{a}_g$ coefficient, a different picture emerges: a globally decreasing trend with a stabilization in the middle of the period—that is, boys turning 11 between 1930 and 1940 did not have more equal access to selective secondary education than the previous cohort (Table 7, cf. Appendix for the method, parameters are calculated from data in Table 6). Even if $\tilde{a}_g$ can potentially vary from 0 to ∞, a value of 1.0 reflects a relatively high inequality for the cohort born before 1910. The overall odds ratio[11] follows the same trend, although the reduction in inequality for the last cohort is not as marked as it is with $\tilde{a}_g$ (Table 7).

While the odds ratio allows for the removal of ambiguities arising from conflicting accounts of the evolution of chance of access and risk of exclusion, knowing inequality within the selection process allows us to distinguish between changes that are attributable to unequal outcomes of micro-social processes and changes attributable to margins variation. Thus, we can see that inequality within the selection process decreases over the sample period, especially for boys reaching 11 years in the 1920s and at the end of the 1940s. In other words, the situation described by Little and Westergaard (1964:312-13) where "the overall expansion of educational facilities has been of greater significance than any redistribution of opportunities," with inequalities probably "shifted to later stages of education where selection would then operate"—that is, a situation that can be understood as unchanged regarding inequality within the selection process—may be discussed somewhat.

We will now compare the results of different analyses of Halsey et al.'s data. Based on the MM calculation, Blackburn and Marsh observed a growing trend of inequality before 1944, which was initially reversed by the Act but then inequality rose again to a higher level than before the Act: The relative shortage of selective places for the last cohort studied, as a result of the "baby boom," they suggested, resulted in greater competition and a reinforcement of the "effective value of social advantage" (Blackburn and Marsh 1991:529). Based on their class categorization, the overall odds ratio indicates a similar pattern of inequality; we can see that this is due to a trend in the inequality within the selection process $\tilde{a}_g$. On the basis of Halsey et al.'s original class categorization, the pattern of change is similar, but less pronounced (Table 8), and we observe no change in the inequality within the selection process for cohorts immediately before and after the Act.

We note that the significant differences in the values of the various indices depend on whether they are based on Floud's classification and data

Table 6. Proportions of Boys in Different Classes Obtaining Education of a Grammar School Type,[a] by Birth Cohort.

% (Eff. Class)	Born Pre-1910	Born 1910–1919	Born 1920–1929	Born Late 1930s
Prof./Manag.	37.0 (378)	44.4 (135)	54.2 (118)	62 (914)
Other nonmanual	12.6 (309)	22.6 (106)	25.7 (105)	34 (874)
Skilled manual	4.8 (834)	10.1 (346)	11.0 (317)	17 (3,647)
Semi- and unskilled	2.3 (437)	4.4 (180)	9.3 (224)	10 (1,786)
Total	11.7 (1,958)	16.5 (767)	19.2 (764)	23 (7,221)

Source: Floud (1954: table 7, 129) and the Crowther Report (1959, II table 9, 130).
[a]Grammar and independent schools.

Table 7. Inequality in the Selection Process[a] $(\tilde{a}_g)$ and Inequality of Opportunity (Overall Odds Ratio), by Birth Cohorts (Boys).

$\tilde{a}_g$ (overall odds ratio)	Born pre-1910	Born 1910–1919	Born 1920–1929	Born late 1930s
Floud's class	1.0 (8.6)	0.85 (6.0)	0.84 (6.0)	0.74 (5.5)

Source: Table 6 data.
[a]For access to grammar and independent schools.

(seven categories) or on Little and Westergaard's grouped classification (three classes) or else, on Halsey et al.'s own classification and data (three classes).[12] Floud's more detailed classification allowed us to distinguish more precisely the advantaged (access rate above average) from the disadvantaged groups (access rate below average), than with Halsey et al.'s classification. Consequently, we will only be able to compare trends in inequality of opportunity, not the extent of inequality, across measures.

The first three cohorts in Halsey et al.'s analysis can be compared to the last three cohorts in Little and Westergaard's analysis, notwithstanding the 3-year difference in dates of birth. However, the comparison with previous analyses of Little and Westergaard's data reveals differences in trends. We observed decreasing inequality within the selection process following the Act, while the situation seems to get worse, after a long, stable period, when we look at results we obtain with Halsey et al.'s data. But, these authors noted an important difference is that they included technical schools in their group of selective schools and Little and Westergaard did not.

Table 8. Inequality in the Selection Process ($\tilde{a}_g$) and Inequality of Opportunity (Overall Odds Ratio), by Birth Cohorts.

$\tilde{a}_g$ (overall odds ratio)	Born 1913–1922	Born 1923–1932	Born 1933–1942	Born 1943–1952
Grammar + techn. Blackburn and Marsh's class	0.54 (3.0)	0.56 (3.2)	0.52 (2.9)	0.60 (3.5)
Grammar + techn. Halsey, Heath and Ridge's class	0.54 (3.0)	0.55 (3.2)	0.55 (3.1)	0.58 (3.3)
Grammar only Halsey, Heath, and Ridge's class	0.58 (3.3)	0.51 (2.9)	0.40 (2.2)	0.39 (2.3)

Source: Table 2 data; Halsey, Heath, and Ridge (1980: table 2.2, 20; table 4.9, 63; and table 4.12, 68).

Significant differences in trends are revealed when we consider grammar schools only. Inequality within the selection process $\tilde{a}_g$ decreases over the three first cohorts and then tends to stabilize (Table 8). The global picture of changes in inequality according to $\tilde{a}_g$ is very coherent: decreasing inequality within the selection process for access to a grammar school education over the first half of the twentieth century, with a slowing of the downward trend, or a stagnation in the 1930s and the beginning of the 1940s, a period of economic recession followed by the Second World War and the exhaustion of the downward trend at the end of the 1950s. The long-term trend of decreasing inequality within the selection process may be interpreted as an effect of the reduction of differences in attitude between social groups toward the value and financial burden of the lengthy studies a grammar school type of education prepared students for.[13] This effect explains the evolutions at issue, such that the Act by itself may not have had any significant impact, an account that is consistent with the fact that it changed little in terms of methods of selection.

These patterns of change in inequality within the selection process can be compared with those observed when we include technical schools in the analysis. The difference in the evolution observed is due to the change in opportunity within the selection process for access to technical schools—variations in the availability of places having, as we know, no direct effect on these changes:[14] increasing inequality in the 1930s and 1940s with a slowing of the upward trend at the end of the 1950s. We suggest that this growing trend of inequality within the selection process for access to technical schools is also an effect of the reduction in differences between social groups regarding their choices of schooling during this period.

The global trends with respect to inequality within the selection process are slightly to the benefit of advantaged social categories when all types of selective schools are considered, but it should still be noted that disadvantaged social categories benefited from a reduction in inequality within the selection process for access to the most selective schools, that is, grammar schools.

To summarize, by separating the effects of inequality within the selection process from the effects of the margins, we obtain a better understanding of the social processes at work. By referring to the evolution of inequality within the selection process, we hypothesize that the development of less socially differentiated attitudes to education from the beginning of the twentieth century—that is, the increased investment by disadvantaged categories in lengthy studies and conversely, the increased investment by advantaged categories in technical studies—underpin two opposing long-term trends that are independent of both the changing number of places in selective schools and the 1944 Act; one is an increase in the inequality within the selection process for access to technical schools and the other is a decrease in the inequality within the selection process for access to a grammar school education, with a trend to stabilization in the last period.

Conclusion

For the purpose of distinguishing the effects of the margins from those of the unequal outcomes of micro-social processes, with respect to inequality of access to a discrete good *G*, the index of inequality within the selection process $\tilde{a}_g$, which we have presented here, exploits the fact that a continuous distribution of relative opportunity within the population can be inferred. The assumption of linearity of the distribution of relative opportunity makes comparisons possible. It allows us to compare the outcomes of micro-social processes, irrespective of the extent of diffusion of *G* in different sets of data.

This index offers a solution to an important class of problem in sociology: explanatory accounts of inequality of access to a discrete good *G* across populations or across time. Such problems have been the objects of regrettably sporadic debate over the last quarter of century. Argument has centered on the issues of margin insensitivity and allocation mechanisms. The aspect of inequality captured by the $\tilde{a}_g$ coefficient that allows a comparison of the outcomes of micro-social processes, irrespective of overall access to *G*, should reduce the confusion surrounding these concepts and put an end to misconceptions about the property of margin insensitivity and the meaning of odds ratios in relation to allocation mechanisms.

Our analysis of the changes in classical measures of inequality with diffusion of G, while inequality within the selection process $\tilde{a}_g$ is held constant, shows that it is not possible to understand this aspect of inequality of opportunity on the basis of any one of these measures. The processes determining access to G are still a black box, but as $\tilde{a}_g$ captures inequality within the selection process, it is meaningful to use this measure to identify factors (micro-social processes unequal outcomes vs. margins) explaining statistical relationships observed at a macro level of analysis.

Appendix

Practical guide to the calculation of $\tilde{a}_g$[15]

1 – Calculate the access rates, $r_{i,}$ to the good G, of the various social subgroups, $C_{i.}$

2 - $\cup C_i = C_g^+$ is defined as the set of C_i subgroups where individuals have a lower than average chance of gaining access to G, i.e. the groups for which r_i is lower than the overall rate of access, x_j. The value m_g is defined as the fraction of the population belonging to social subgroup C_g^+, r_g is the access rate to G of members of C_g^+.

$$\text{Calculate } a_g = \frac{2 \times m_g \times (x_j - r_g)}{(1 - x_j) \times x_j}$$

Case 1 General case: $\tilde{d}(x, C_g^+)$ intersects the two sides of the square where $\tilde{f}(x, C_g^+)$ is traced:

$$\frac{a_g}{2} \leq m_g \leq 1 - \frac{a_g}{2} \quad \text{then} \quad \tilde{a}_g = a_g = \frac{2 \times m_g \times (x_j - r_g)}{(1 - x_j) \times x_j}$$

Case 2 $\tilde{d}(x, C_g^+)$ intersects the base and one side of the square where $\tilde{f}(x, C_g^+)$ is traced:

$$m_g \leq \inf\left(\frac{a_g}{2}, 1 - \frac{a_g}{2}\right) \quad \text{then} \quad \tilde{a}_g = 2m_g\left[\frac{1 - \sqrt{r_g}}{1 - x_j}\right]^2$$

Case 3 $\tilde{d}(x, C_g^+)$ intersects the top and one side of the square where $\tilde{f}(x, C_g^+)$ is traced:

$$m_g \geq \sup\left(\frac{a_g}{2}, 1-\frac{a_g}{2}\right) \quad \text{then} \quad \tilde{a}_g = 2\left[\frac{\sqrt{1-m_g}-\sqrt{1-x_j-m_g+r_g m_g}}{x_j}\right]^2$$

Case 4 $\tilde{d}(x, C_g^+)$ intersects the base and the top of the square where $\tilde{f}(x, C_g^+)$ is traced:

$$1-\frac{a_g}{2} \leq m_g \leq \frac{a_g}{2} \quad \text{then} \quad \tilde{a}_g = \frac{1}{2}\left[\frac{1}{\sqrt{r_g m_g}+\sqrt{1-x_j-m_g+r_g m_g}}\right]^2$$

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Notes

1. “State secondary” education was then limited to those pupils who were admitted to aided/maintained grammar schools, those who attended junior technical colleges, and to “the great majority who ‘stayed on’ in the senior departments of the elementary schools until they reached the leaving age of 14” (Floud 1954:101).
2. The use of general records, teachers’ assessments, and traditional examinations increased in place of standardized ability tests. Blackburn and Marsh (1991: 529) also mention that, from 1932 until the passage of the 1944 Act, the “Special Place system” replaced the “Free Place system”: After a pupil was successful in the scholarship competition, his or her parents’ income was taken into account before either a free or subsidized place was awarded.
3. For the last cohort (born in the late 1930s), the figures for boys’ secondary schooling were derived from the Crowther Report’s sample analysis of national service recruits; and “the figures of girls’ secondary schooling from those from the boys, adjusted to allow for the slight sex differences in educational experiences at these stage indicated by the *Early Leaving Report*” (Little and Westergaard 1964:303).

4. According to Boudon (1974:145), a general overall increase in school attendance rates may have the consequence of reducing inequality of educational opportunity. This hypothesis relied on ratios of the percentages accessing a given educational level within each social background category, but Boudon also pointed out that calculating the percentage point differences led to an opposing diagnosis.
5. Formed by subtracting the proportion selected in the bottom class from that in the top class, the difference of proportions, *d*, can be interpreted as the unstandardized regression coefficient between two dummy variables (Blackburn and Marsh 1991:519).
6. $\varphi^2 = \frac{\chi^2}{N}$.
7. The odds is the probability of an event occurring divided by the probability of the event not occurring for example, gaining access to G divided by being excluded from G: $p_1/(1 - p_1)$. An odds ratio is the odds of a particular outcome in one group divided by the odds for the same outcome in the other group. If p_1 is the probability of the outcome in group 1, and p_2 is the probability of the outcome in group 2, we have odds ratio $= \frac{p_1/(1-p_1)}{p_2/(1-p_2)}$.
8. Marshall and Swift (1999:248) were thus right when they responded to critics of a margin-insensitive measure such as the odds ratio with the argument that it is pointless to assert that distributional outcomes "must" take priority regarding the concept of inequality, because inequality is always inequality of something and in the case of education: "it is appropriate to focus on the distribution of chances of achieving and avoiding unequal levels of education, rather than on the distribution of education as such, when it is people's relative amount of education that is relevant to how their education converts into occupational, and hence class, outcomes."
9. It is worth noting here that the defined criterion for distinguishing between the advantaged groups and the disadvantaged groups is not chosen arbitrarily but has meaning regarding the shape of the groups' distributions of opportunities (see Properties of the $(\tilde{a}_i)$ coefficients subsection 'c').
10. Note that a possible change in the respective compositions of C_g^+ and C_g^- from one context to another does not matter for comparisons of this (overall) inequality.
11. Comparing all the boys from the advantaged categories—that is, those with an above average rate of access to selective schools—and all the boys from the disadvantaged categories—that is, those with a below average rate of access to selective schools.
12. Halsey, Heath, and Ridge (1980:72, note 15) noted: "The reader may be worried that while the trend is now the same, the absolute figures still differ markedly from those of Little and Westergaard. However, the differences are largely due

to the differing classifications of social class used. If we attempt to reclassify our data in broadly the same way as Little and Westergaard, our results become quite closely in line with theirs."

13. This hypothesis is confirmed by the substantial "refusal rate"—more than 50 percent in the early 1920s—of the free places offered in secondary schools, which was mainly imputable to the issue of sacrificing potential earnings (Floud, Halsey, and Martin 1957:34, 117, 147; Lindsay 1926:11, 43, 199). This explanation is also consistent with the analysis of students who gained access to secondary schools. According to Halsey et al. (1980:146), insofar as cultural differences were in operation, "for those who passed through the British schools from the 1920s to the 1960s, it took the form of differential access to selective schooling, and not of differential performance."
14. Halsey et al. note that the expansion of the grammar schools in the postwar period was largely at the expense of the technical schools, which saw a steady decline in places over the two last cohorts. Technical school places grew from 13.6% to 17.6% of total secondary school places before falling to 10.7% then 6.2%, whereas the proportion of grammar school places grew steadily over the same period, with the successive proportions of 10.9%, 13.9%, 20.6%, and 21.2% (Halsey et al. 1980:67, table 4.12, 68).
15. According to Bulle (2009: 583-588).

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